People's Democratic Republic of Algeria
Ministry of Higher Education and Scientific Research

University of Batna 2 — Common Core of Science and Technology
Faculty of Technology — Engineering Section

Module: Algebra 1 (Algèbre 1)

# Tutorial Session N°1

**Exercise n°1**

Let $U = R$, be the universal set, $A = \{x \in R \,/\, 0 < x \leq 3\}$, and $B = \{x \in R \,/\, 2 \leq x < 4\}$.

Find $A \cup B$, $A \cap B$, $B - A$, and $\bar{A}$.

**Exercise n°2**

a. Let $U = R$, and $A_i = ]-i, i[$ for each positive integer $i$. Find $\cup_{i=1}^{\infty} A_i$.
b. Let U=R, and $A_i = \left]0, \frac{1}{i}\right[$ for each positive integer $i$. Find $\cap_{i=1}^{\infty} A_i$.

**Exercise n°3**

a. Let U=R, and $A_i = \left[\frac{1}{i+1}, \frac{1}{i}\right[$ for each positive integer $i$. Show that the collection $\{A_1, A_2, \dots\}$ forms a partition of $]0,1[$.
b. Let U=R, and $A_i = [-i, i[$ for each positive integer $i$. Explain why the collection $\{A_1, A_2, \dots\}$ is not a partition of $]-\infty, +\infty[$.
c. Let U=R, and $A_i = [i-1, i[$ for each positive integer $i$. Explain why the collection $\{A_1, A_2, \dots\}$ is not a partition of $]0, +\infty[$.

**Exercise n°4**

a. Let $A = \{7,10\}$. Find $P(A)$.
b. Let $A = \{a, b, c, d\}$. Find $P(A)$.
c. Let $A = \{a, b, \}$ and $A = \{a, d\}$.
   i Find $A \times B$.
   ii Find $B \times A$.

**Exercise n°5**

a. Prove the distributive law $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
b. Prove the following: for all sets $A$ and $B$, if $A \subseteq B$, then $\bar{B} \subseteq \bar{A}$

**Exercise n°6**

a. Let $f: Z \to Z$ be the function defined by $n \to 3n$.
   i Find the range of $f$.
   ii Find the preimages of 12, 18, and 20.
b. Let $f: Z \to Z$ be the function defined by $n \to n + 4$.
   i Find the range of $f$.
   ii Find the preimages of -5, 8, and 17.

**Exercise n°7**

Let $f: Z \to Z$ be the function defined by $f(n) = 5n + 4$. Show that $f$ is one-to-one.

**Exercise n°8**

Let $f: Z \to Z$ be the function defined by $f(n) = n^4$. Show that $f$ is not one-to-one.

**Exercise n°9**

Let $f: Z \to Z$ be the function defined by $f(n) = 2^n$. Show that $f$ is not onto.

**Exercise n° 10**
Let $f: Z \to Z$ be the function defined by $f(n) = -n$. Show that $f$ is onto.
**Exercise n°11**
Let $f: Z \to 2Z$ be the function defined by $f(n) = 2n$. Show that $f$ is onto. $2Z$ is the set of all even integers. Show that $f$ is a one-to-one correspondence.
**Exercise n°12**
Find the inverse function of the function in the previous exercise.
**Exercise n°13**
Let $f: Z^+ \to Z^-$ be defined by $f(n) = -n$. $Z^+$ is the set of positive integers, and $Z^-$ is the set of negative integers. Show that $f$ is one-to-one correspondence.
**Exercise n° 14**
Find the inverse function of the function in the previous exercise.
**Exercise n°15**
Let $f: R \to R$ be defined by $f(x) = |x|$, and $g: R \to R$ be defined by $g(x) = x + 1$. Find the composition $g \circ f$, then find $f \circ g$. Determine if $f \circ g = g \circ f$.
**Exercise n°16**
Let $B$ be the set of all finite strings of 0's and 1's. let $f: B \to B$ be defined by $f(b) = \overleftarrow{b}$, where $\overleftarrow{b}$ is the string $b$ in reverse order.
Let $g: B \to B$ be defined by $g(b) = 1b1$ the string $b$ with 1 attached in front of $b$ and after $b$. For example, if $b = 01011$, then $1b1 = 1010111$. Find the composition $g \circ f$, then find $f \circ g$. Determine if $f \circ g = g \circ f$.
**Exercise n°17**
Let $f: ]1, \infty[ \to ]0,1[$ be defined by $f(x) = \frac{1}{x}$. Show that f is a bijection.
**Exercise n°18**
Let $B$ be the set of all finite strings of 0's and 1's. let $1B1$ be the set of all finite strings of 0's and 1's that begin and end with a 1. Prove that $B$ and $1B1$ have the same cardinality.
**Exercise n°19**
Define a relation $\mathcal{R}$ on $Z^+$ as follows. For positive integers $m$ and $n$, $m\mathcal{R}n$ just in case $m|n$ (that is $m$ divides $n$). Determine if $3\mathcal{R}(12)$, $4\mathcal{R}(-16)$, $3\mathcal{R}(11)$.
**Exercise n°20**
Let $B$ be the set of all finite strings of 0's and 1's. Define a relation on $B$ as follows:
For strings $b_1$ and $b_2$, $b_1\mathcal{R}b_2$ just in case the number of 1's in $b_1$ is the same as the number of 1's in $b_2$. Determine if $(0101)\mathcal{R}(1100)$, $(0101)\mathcal{R}(1000)$, $(1101010)\mathcal{R}(101101)$.
**Exercise n°21**
Consider the "divides" relation $\mathcal{R}$ on $Z^+$ defined as follows:
For positive integers $m$ and $n$, $m\mathcal{R}n$ just in case $m|n$. Determine if $\mathcal{R}$ is reflexive, symmetric, or transitive.
**Exercise n°22**
Consider the relation $\mathcal{R}$ on $B$ defined as follows:
For strings $b_1$ and $b_2$, $b_1\mathcal{R}b_2$ just in case the number of 1's in $b_1$ is the same as the number of 1's in $b_2$. Determine if $\mathcal{R}$ is reflexive, symmetric, or transitive.
**Exercise n° 23**
Consider the relation $\mathcal{R}$ on $Z$ defined as follows:
For integers $m$ and $n$; $m\mathcal{R}n$ just in case $m = n$ or $m = -n$. Show that $\mathcal{R}$ is an equivalence relation.
**Exercise n°24**
Consider the relation $\mathcal{R}$ on $Z$ defined as follows:
For integers $m$ and $n$; $m\mathcal{R}n$ just in case $3|(m^2 - n^2)$. Show that $\mathcal{R}$ is an equivalence relation.

**Exercise n°25**

Let $S$ be the set of all finite strings of 0's and 1's of length 2 or more. Consider the relation $\mathcal{R}$ on $S$ defined as follows:

For strings $b_1$ and $b_2$ in $S$, $b_1\mathcal{R}b_2$ just in case the first two characters of $b_1$ are the same as the first two characters of $b_2$. Show that $\mathcal{R}$ is an equivalence relation.

**Exercise n°26**

Let $S$ be the set of all finite strings of 0's and 1's of length 2 or more. Let $\mathcal{R}$ be the equivalence relation on $S$ defined as follows:

For strings $b_1$ and $b_2$ in $S$, $b_1\mathcal{R}b_2$ just in case the first two characters of $b_1$ are the same as the first two characters of $b_2$. Find the partition of $S$ induced by $\mathcal{R}$.

**Exercise n°27**

Let $B$ be the set of all finite strings of 0's and 1's. let $\mathcal{R}$ be the equvalence relation on $B$ defined as follows:

For strings $b_1$ and $b_2$; $b_1\mathcal{R}b_2$ just in case the number of 1's in $b_1$ is the same as the number of 1's in $b_2$. Find the partition of $B$ induced by $\mathcal{R}$.

**Exercise n°28**

Let $\mathcal{R}$ be the equivalence relation on $Z$ defined as follows:

For integers $m$ and $n$; $m\mathcal{R}n$ just in case $m = n$ or $m = -n$. Find the partition of $Z$ induced by $\mathcal{R}$.

**Exercise n°29**

Let B be the set of all finite strings of 0's and 1's. for $i \geq 0$, let $A_i$ be the set of all strings in $B$ that have length $i$. Then $P = \{A_i | i \geq 0\}$ is a partition of $B$. Determine if $1100$, $10101$, $0001011$ lie in $[11011]$.

**Additional exercises**

**Exercise n°1**

What are the possible values of $n$ given this set notation: $\{n \in Z^+ | n \text{ is a factor of } 8\}$.

**Exercise n°2**

Identify each of the following as true or false.

$\{3\} \in \{1, 3, 5, 7\}$

$\{3\} \subseteq \{1, 3, 5, 7\}$

$\{3\} \in \{\{1\}, \{3\}, \{5\}, \{7\}\}$

$\{3\} \subseteq \{\{1\}, \{3\}, \{5\}, \{7\}\}$

**Exercise n°3**

For all sets $A$, $B$, and $C$, prove that $A - (A \cap B) = A - B$. List the name of each law used.

**Exercise n°4**

Let set $A$ be a set of all university of Batna 2 (UB2) employees and $B$ is the set of teachers. Describe the following statements.

$A \cap B$

$A \cup B$

$A - B$

$B - A$

**Exercise n°5**

Illustrate the following using Venn Diagrams.

a. $(A \cap B) - A$
b. $(A - B) \cup (B - A)$

**Exercise n°6**

What is the power set of $\{a, b, c\}$