

People's Democratic Republic of Algeria  
Ministry of Higher Education and Scientific Research

University of Batna 2                      Common Core of Science and Technology  
Faculty of Technology                      Engineering Section

Module: Algebra 2 (Algèbre 2)

**Tutorial Session N°1**

**Exercise n°1**

Solve the following system of linear equations using substitution.

$$y = 4x - 3$$

$$y = 3x - 1$$

**Exercise n°2**

Find where functions intersect.

$$3x - 4y = 2$$

$$4x + 3y = 14$$

**Exercise n°3**

Solve the following system of linear equations using elimination.

a. 
$$\begin{aligned} 3x+4y &= -5 \\ x-5y &= -8 \end{aligned}$$

b. 
$$\begin{aligned} x+2y &= 8 \\ 3x-4y &= 4 \end{aligned}$$

c. 
$$\begin{aligned} x-3y &= -7 \\ 2x-6y &= 7 \end{aligned}$$

**Exercise n°4**

Solve the following system of linear equations using elimination.

$$3x + 2y = 1$$

$$2x + 3y = 1$$

**Exercise n°5**

Solve the following system of three linear equations by elimination.

a. 
$$\begin{aligned} x+2y-z &= 1 \\ 2x-y+z &= 6 \\ 2x-y-z &= 0 \end{aligned}$$

d. 
$$\begin{aligned} x+2y+3z &= 6 \\ 2x-3y+2z &= 14 \\ 3x+y-z &= -2 \end{aligned}$$

b. 
$$\begin{aligned} 2x-3y+6z &= 3 \\ x+2y-4z &= 5 \\ 3x+4y-8z &= 7 \end{aligned}$$

e. 
$$\begin{aligned} x+2y &= 10 \\ 2x-2y &= -4 \\ 3x+5y &= 26 \end{aligned}$$

c. 
$$\begin{aligned} 2x+y-z &= -2 \\ 3x+2y+3z &= 21 \\ 7x+4y+z &= 17 \end{aligned}$$

f. 
$$\begin{aligned} x+2y &= 10 \\ 2x-2y &= -4 \\ 3x+5y &= 20 \end{aligned}$$

**Exercise n°6**

Solve the following system. (2 equations in 3 unknowns)

a. 
$$\begin{aligned} 2x-2y+5z &= 14 \\ x+4y-3z &= -2 \end{aligned}$$

b. 
$$\begin{aligned} 4x-2y+6z &= 5 \\ 2x-y+3z &= 4 \end{aligned}$$

c. 
$$\begin{aligned} x+2y-3z &= -4 \\ 2x+y-3z &= 4 \end{aligned}$$

**Exercise n°7**

a. Let  $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 5 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 0 \end{bmatrix}$ . Find  $A+B$ ,  $A-B$ ,  $3 \cdot A$ ;  $5 \cdot B$ ; and  $3 \cdot A + 5 \cdot B$

b. Suppose  $r=2$ ; and  $A = \begin{bmatrix} 1 & 3 & -5 \\ 2 & 4 & 6 \\ 3 & -6 & 2 \end{bmatrix}$ . Find  $r \cdot A$ ;  $A^T$ .

c. Let  $A = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 4 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 6 & -5 & 4 \\ 3 & 0 & -8 \end{pmatrix}$ . Find  $(2A - 3B)^T$

**Exercise n°8**Find the products  $AB$  and  $BA$  of the following matrices if they exist.

a.  $A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 4 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 1 & 2 & -1 \\ 3 & 0 & 0 & 4 \\ -1 & 1 & 5 & 1 \end{bmatrix}$

b.  $A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 4 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 4 \\ 0 & 7 \\ -1 & 2 \end{bmatrix}$

c.  $A = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ ,  $B = [1 \quad 6 \quad 10]$

d.  $A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 1 & 4 \end{pmatrix}$ ,  $B = \begin{pmatrix} -2 & 5 \\ 4 & -3 \\ 2 & 1 \end{pmatrix}$

e.  $A = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{pmatrix}$

f.  $A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$ ,  $B = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$

**Exercise n°9**

Solve the following system of equations using the Gauss-Jordan method.

a.  $x + y = 5$

$x - y = -1$

b.  $3x - 4y = 1$

$5x + 2y = 19$

$$2x + y - z = 8$$

c.  $-3x - y + 2z = -11$   
 $-2x + y + 2z = -3$

**Exercise n°10**

Solve the following system of equations using the Row Echelon Form (Gaussian Elimination).

$$x - 2y + 3z = 9$$

a.  $-x + 3y = -4$   
 $2x - 5y + 5z = 17$

$$2x + 4y + z = -4$$

b.  $2x - 4y + 6z = 13$   
 $4x - 2y + z = 6$

$$3x - 2y + 4z = 1$$

c.  $x + y - 2z = 3$   
 $2x - 3y + 6z = 8$

$$x + 2y - z = 5$$

d.  $4x - y + 5z = 11$   
 $5x - 8y + 13z = 7$

**Exercise n°11**

Solve the following system of equations using the Reduced Row Echelon form (also called Gauss–Jordan elimination).

$$x - 2y + 3z = 9$$

a.  $-x + 3y = -4$   
 $2x - 5y + 5z = 17$

$$y + z - 2w = -3$$

b.  $x + 2y - z = 2$   
 $x + 4y + z - 3w = -2$   
 $x - 4y - 7z - w = -19$

c.  $-x + 2y = 4$   
 $2x - 4y = 3$

d.  $x + y - 5z = 3$   
 $x - 2z = 1$   
 $2x - y - z = 0$

**Exercise n°12**

Prove the following properties.

- $A + I = A$
- $A + (-A) = I$
- $AI = A$
- $AA^{-1} = I$

**Exercise n°13**

Let  $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ , find the inverse of this matrix  $A^{-1}$ , using the RRE form.

**Exercise n°14**

Solve the following system of equations, using the RRE form.

$$2x + 3y = 0$$

$$x + 4y = 3$$

**Exercise n°15**

Solve the following system of equations using the inverse of a square matrix.

$$3x + 2y + 2z = -1$$

$$2x + 2y + 2z = 2$$

$$4x + 4y + 3z = 0$$

**Exercise n°16**

Find the determinant of the following matrices.

a.  $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

b.  $\begin{bmatrix} -4 & 5 \\ -3 & -1 \end{bmatrix}$

c.  $\begin{bmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 4 & 0 & 1 \end{bmatrix}$

d.  $\begin{pmatrix} 1 & -2 & 3 & 0 \\ -1 & 1 & 0 & 2 \\ 0 & 2 & 0 & 3 \\ 3 & 4 & 0 & 2 \end{pmatrix}$

e.  $\begin{pmatrix} 4 & 0 & 0 & 0 \\ 1 & -4 & 0 & 0 \\ 2 & 1 & -1 & 0 \\ 6 & -2 & 3 & -1 \end{pmatrix}$

f.  $\begin{pmatrix} 3 & 6 & -5 & 4 \\ -2 & 0 & 6 & 0 \\ 1 & 1 & 2 & 2 \\ 0 & 3 & -1 & -1 \end{pmatrix}$

**Exercise n°17**

Let  $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 5 & 8 & 9 \end{pmatrix}$ . Find.

a.  $|A|$

b.  $adj(A)$

c.  $A^{-1}$  using  $adj(A)$

**Exercise n°18**

Solve the following systems using Cramer Rule.

a.  $4x - 2y = 10$

$3x - 5y = 11$

b.  $4x - 2y + 3z = -2$

$2x + 2y + 5z = 16$

$8x - 5y - 2z = 4$