corrigé Type Rattrapage P_1 2023/2024
Ingenieurs

question de cours

$\vec{v} = \dfrac{d\vec{OM}}{dt}$ (0,25) avec $\vec{OM} = r\vec{e_r} + z\vec{e_z}$ (0,25)

$\vec{v} = \dot{r}\vec{e_r} + r\dot{\theta}\vec{e_\theta} + \dot{z}\vec{e_z}$ (1pt)

avec $\begin{cases} \dfrac{d\vec{e_r}}{dt} = \dfrac{d\vec{e_r}}{d\theta}\cdot\dfrac{d\theta}{dt} = \dfrac{d\vec{e_r}}{d\theta}\cdot\dot{\theta} \\ \dfrac{d\vec{e_\theta}}{dt} = \dfrac{d\vec{e_\theta}}{d\theta}\cdot\dfrac{d\theta}{dt} = \dfrac{d\vec{e_\theta}}{d\theta}\cdot\dot{\theta} \\ \dfrac{d\vec{e_z}}{dt} = 0 \end{cases}$

et $\begin{cases} \vec{e_r} = \cos\theta\,\vec{i} + \sin\theta\,\vec{j} \\ \vec{e_\theta} = -\sin\theta\,\vec{i} + \cos\theta\,\vec{j} \end{cases}$ (0,25pt)

d'où $\begin{cases} \dfrac{d\vec{e_r}}{d\theta} = \dfrac{d}{d\theta}(\cos\theta\,\vec{i} + \sin\theta\,\vec{j}) = -\sin\theta\,\vec{i} + \cos\theta\,\vec{j} = \vec{e_\theta} \\ \dfrac{d\vec{e_\theta}}{d\theta} = \dfrac{d}{d\theta}(-\sin\theta\,\vec{i} + \cos\theta\,\vec{j}) = -(\cos\theta\,\vec{i} + \sin\theta\,\vec{j}) = -\vec{e_r} \end{cases}$

d'où $\begin{cases} \dfrac{d\vec{e_r}}{dt} = +\dot{\theta}\vec{e_\theta} \\ \dfrac{d\vec{e_\theta}}{dt} = -\dot{\theta}\vec{e_r} \\ \dfrac{d\vec{e_z}}{dt} = 0 \end{cases}$ (0,25pt)

$\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d^2\vec{OM}}{dt^2} = (\ddot{r} - r\dot{\theta}^2)\vec{e_r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{e_\theta} + \ddot{z}\vec{e_z}$

(0,25) (1pt)

exo 1 $\vec{v} = \vec{i} + (2t-3)\vec{j}$

1) $|\vec{v}| = \sqrt{1+(2t-3)^2} = \sqrt{4t^2-12t+10}$ m/s (0,5pt)

2) $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\vec{i} + (2t-3)\vec{j}\right) = 2\vec{j}$ (1pt) (0,5pt)

$|\vec{a}| = \sqrt{(2)^2} = 2$ m/s^2 (0,5pt)

3) $\vec{OM} = \int \vec{v}\,dt = \int \left[\vec{i} + (2t-3)\vec{j}\right] dt$ (0,5pt)

$\vec{OM} = (t + C_1)\vec{i} + (t^2 - 3t + C_2)\vec{j}$ (0,5pt)

CI → M(0,3) à t=0 $\begin{cases} x_0 = 0 = t + C_1 \Rightarrow C_1 = 0 \\ y_0 = 3 = t^2 - 3t + C_2 \Rightarrow C_2 = 3 \end{cases}$ (0,25pt) (0,25pt)

$\vec{OM} = t\vec{i} + (t^2 - 3t + 3)\vec{j}$ (0,5pt)

4) $\begin{cases} x = t & (1) \\ y = t^2 - 3t + 3 & (2) \end{cases}$ (1) ds (2) ⇒ $y = x^2 - 3x + 3$ (2pt)

l'équation de la trajectoire

5) $\vec{a} = \vec{a}_T + \vec{a}_N$ (repère de Fresnet) (0,5)

$\begin{cases} a_T = \frac{dv}{dt} \quad (0,5) \\ a_N = \frac{v^2}{R} \quad (0,5) \end{cases}$

$\Rightarrow a_T = \frac{d}{dt}\left(\sqrt{4t^2-12t+10}\right)$

$a_T = \frac{8t-12}{2\sqrt{4t^2-12t+10}}$

$a_T = \frac{4t-6}{\sqrt{4t^2-12t+10}}$ (0,5pt)

$a^2 = a_T^2 + a_N^2 \Rightarrow a_N^2 = a^2 - a_T^2 \Rightarrow a_N = \sqrt{a^2 - a_T^2}$ (0,5)

$$a_N^2 = (2)^2 - \frac{(4t-6)^2}{(4t^2-12t+10)} = \frac{4}{4t^2-12t+10}$$

$$\boxed{a_N = \frac{2}{\sqrt{4t^2-12t+10}}}$$ (0,5pt)

$$a_N = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{a_N}$$

$$R = \frac{4t^2-12t+10}{\frac{2}{\sqrt{4t^2-12t+10}}} = \frac{(4t^2-12t+10)^{3/2}}{2} \text{ m}$$ (0,5pt)

ou $R = \frac{2}{\sqrt{2}}(2t^2-6t+5)^{3/2}$

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exo2 PFD : $\sum \vec{F}_{ext} = m\vec{a} \Rightarrow \vec{F}_1 + \vec{F}_2 = m\vec{a}$ (1pt)

(1pt) $a_1 \sin\omega t\,\vec{i} + a_2 \cos\omega t\,\vec{j} = m\vec{a} = m\frac{d\vec{v}}{dt}$

$\vec{v} = \int \vec{a}\,dt = \int \left(\frac{a_1}{m}\sin\omega t\,\vec{i} + \frac{a_2}{m}\cos\omega t\,\vec{j}\right)dt$ (0,5) (0,5pt)

$\vec{v} = \left(-\frac{a_1}{m\omega}\cos\omega t + C_1\right)\vec{i} + \left(\frac{a_2}{m\omega}\sin\omega t + C_2\right)\vec{j}$ (1pt)

à $t=0$ $\vec{v}=0 \Rightarrow \begin{cases} -\frac{a_1}{m\omega}\cos 0 + C_1 = 0 \Rightarrow C_1 = \frac{a_1}{m\omega} \\ \frac{a_2}{m\omega}\sin 0 + C_2 = 0 \Rightarrow C_2 = 0 \end{cases}$ (0,5) (0,5)

$$\vec{v} = \left(\frac{a_1}{m\omega} - \frac{a_1}{m\omega}\cos\omega t\right)\vec{i} + \frac{a_2}{m\omega}\sin\omega t\,\vec{j}$$

$$\vec{v} = \frac{a_1}{m\omega}(1-\cos\omega t)\,\vec{i} + \frac{a_2}{m\omega}\sin\omega t\,\vec{j}$$ (1pt)