Model "Analyse 1" Calculus 1.

Final Exam
Standard Answer

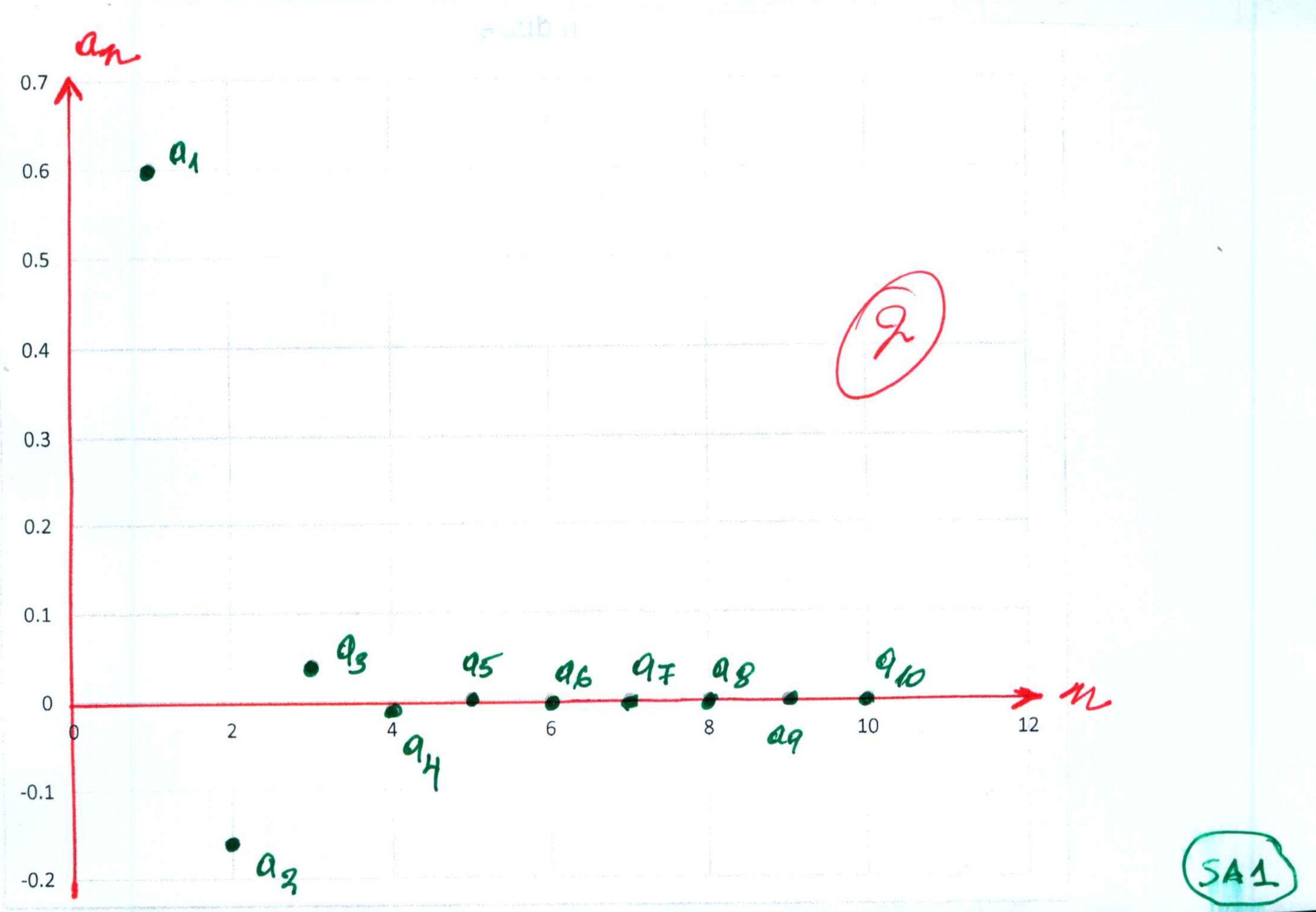
Exercise n°1

We are given that

$a_1 = \frac{3}{5}$ $a_2 = -\frac{4}{25}$ $a_3 = \frac{5}{125}$ $a_4 = \frac{-6}{625}$, and $a_5 = \frac{7}{3125}$

It has to be noticed that the numerators of these fractions start with the number 3, and increase by 1. So the n^{th} term will be $n+2$. The denominators are the powers of 5, so the denominator of a_n is 5^n. ~~the~~ The signs of terms are alternatively positive and negative. so we use $(-1)^{n-1}$.

Therefore $\boxed{a_n = (-1)^{n-1} \frac{n+2}{5^n}}$ 2



SA1

Exercise n°2 (7.5 points)

a).
$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \quad (1) \qquad f(x) = 3x^3 + 2x - 1.$$

$$= \lim_{h\to 0} \frac{[3(x+h)^3 + 2(x+h) - 1] - (3x^3 + 2x - 1)}{h}$$

$$= \lim_{h\to 0} \frac{3(\cancel{x^3} + 3x^2h + 3xh^2 + h^3) + \cancel{2}(\cancel{x}+h) \cancel{-1} \cancel{-3x^3} \cancel{-2x} \cancel{+1}}{h}$$

$$= \lim_{h\to 0} \frac{9x^2\cancel{h} + 9xh^{\cancel{2}} + 3h^{\cancel{3}\,2} + 2\cancel{h}}{\cancel{h}}$$

$$= \lim_{h\to 0} 9x^2 + 9xh + 3h^2 + 2$$

$$f'(x) = 9x^2 + 2. \quad (2)$$

$$f'(1) = 9(1)^2 + 2 = 11 \quad (0,5)$$

$$f'(2) = 9(2)^2 + 2 = 38 \quad (0,5)$$

$$f'(3) = 9(3)^2 + 2 = 83. \quad (0,5)$$

b).
$$x^2 + y^2 = 25$$

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(25) \quad (1)$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 0$$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

or $\frac{dy}{dx} = \frac{-x}{y}$ (1)

at the point (3,4), we have

$$\frac{dy}{dx} = -\frac{3}{4} \quad (1)$$

therefore the equation of the tangent line is

$$y - 4 = -\frac{3}{4}(x - 3) \quad (1)$$

SA2

Exercise n°3

a. $f(x) = e^{-x}$ 5 pts $[0,2]$

$$R_n = \Delta x \left[f(a+\Delta x) + f(a+2\Delta x) + \dots + f(a+n\Delta x) \right]$$

$$L_n = \Delta x \left[f(a) + f(a+\Delta x) + \dots + f(a+(n-1)\Delta x) \right]$$

$$M_n = \Delta x \left[f\left(a+\frac{\Delta x}{2}\right) + f\left(a+\frac{3}{2}\Delta x\right) + \dots + f\left(a+\frac{(n-1)}{2}\Delta x\right) \right]$$

ou ou ou

$$\Delta x = \frac{2-0}{4} = \frac{1}{2} = 0,5$$ 0,5

$$R_4 = 0,5 \left[f(0+0.5) + f(0+1) + f(0+1,5) + f(0+2) \right]$$ 0,5

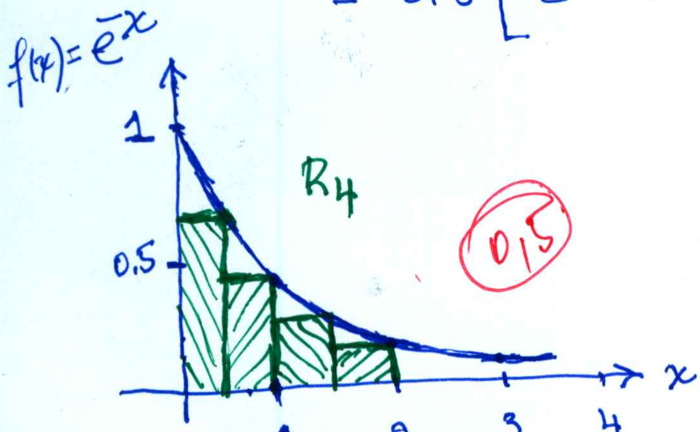
$$= 0.5 \left[e^{-0,5} + e^{-1} + e^{-1.5} + e^{-2} \right] = 0.666438$$ 0,5

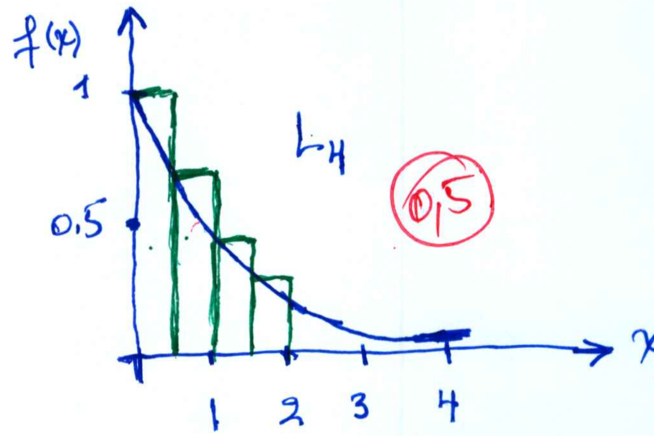
$$L_4 = 0.5 \left[f(0) + f(0+0,5) + f(0+1) + f(0+1,5) \right]$$ 0,5

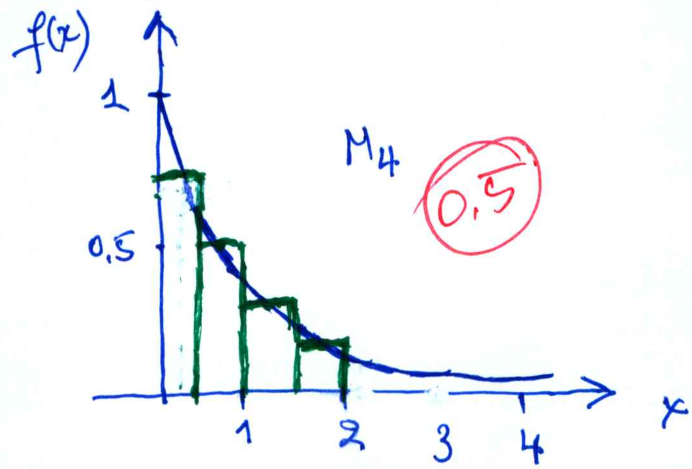
$$= 0.5 \left[e^{-0} + e^{-0.5} + e^{-1} + e^{-1.5} \right] = 1.09877$$ 0,5

$$M_4 = 0,5 \left[f\left(0+\frac{0.5}{2}\right) + f(0+0.75) + f(0+1.25) + f(0+1.75) \right]$$ 0,5

$$= 0,5 \left[e^{-0.25} + e^{-0.75} + e^{-1.25} + e^{-1.75} \right] = 0.855723$$ 0,5







b. Find $\int t^2 e^t dt$ 3.5 /4

let $u = t^2 \quad dv = e^t dt$

$du = 2t\,dt \quad v = e^t$ 1

so $\int t^2 e^t dt = t^2 e^t - 2\underbrace{\int t e^t dt}$ 0.5

let $u = t \quad dv = e^t dt$

$du = dt \quad v = e^t dt$ 1

so $\int t^2 e^t dt = t^2 e^t - 2\left[t e^t - \int e^t dt\right]$ 0.5

$= t^2 e^t - 2\left[t e^t - e^t + c\right]$

$= t^2 e^t - 2te^t + 2e^t - 2C$ 0.5

$$\boxed{\int t^2 e^t dt = t^2 e^t - 2te^t + C + 2e^t}$$

SA4