

People's Democratic Republic of Algeria  
Ministry of Higher Education and Scientific Research  
University of Batna 2                      Common Core of Science and Technology  
Faculty of Technology                      Engineering Section  
Module: Calculus 1 (Analyse 1)  
**Tutorial Session N°3**

**Exercise n°1**

Investigate the properties of the following functions.

- a.  $g(x) = \frac{1}{x}$
- b.  $h(x) = \cos x$

**Exercise n°2**

Write the following as a function composition.

- a.  $h(x) = (6x - 13)^8$
- b.  $h(x) = \ln(\tan 2x - 3)$

**Exercise n°3**

- a. Find the average rate of change of  $f(x) = e^x$  on:
  - $[1, 2]$
  - $[1, 1.1]$
  - $[1, 1.01]$
  - Estimate the instantaneous rate of change at  $x = 1$
- b. Find the average rate of change of  $f(x) = \sin x$  on:
  - $[0, 1]$
  - $[0, 0.1]$
  - $[0, 0.01]$
  - Estimate the instantaneous rate of change at  $x = 0$

**Exercise n°4**

Investigate the following limits:

- a.  $\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{x - 2}$
- b.  $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x}$

**Exercise n°5**

Compute the following limits:

- a.  $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x}$
- b.  $\lim_{x \rightarrow \infty} \frac{3x^5 + 7x - 1}{5x^4 - 9}$

**Exercise n°6**

Use the formal definition of the limit to prove:

- a.  $\lim_{x \rightarrow 4} (-2x + 13) = 5$
- b.  $\lim_{x \rightarrow 2} \frac{1}{x} = \frac{1}{2}$

**Exercise n°7**

Determine the points at which each function is discontinuous.

- $f(x) = \frac{1}{\sqrt{x-1}}$
- $f(x) = \ln x$
- $f(x) = \frac{\sin x}{x}$

**Exercise n°8**

Apply the intermediate value theorem to the function  $f(x) = \frac{1}{x}$  on the interval  $[1, 5]$ .

**Exercise n°9**

Use the limit definition of the derivative to compute the derivative function  $f'(x)$  for:

- a.  $f(x) = \frac{1}{x}$
- b.  $f(x) = \sqrt{x}$

**Exercise n°10**

Use the limit definition of the limit to prove:

$$\frac{d}{dx} \left[ \frac{1}{x^2} \right] = -\frac{2}{x^3}$$

**Exercise n°11**

Use the Power Rule to compute the derivative of the following:

- a.  $f(x) = 3x^6 - 2x^8 - 4x^2 + 5x - 2$
- b.  $g(x) = 2x^{\frac{7}{2}} - x^{\frac{1}{9}} - 4$
- c.  $h(x) = \frac{3}{x} + 5x^{-3} + x^{-\frac{2}{3}}$

**Exercise n°12**

Calculate the derivative of the following functions:

- a.  $h(x) = \sqrt{x} \left[ \frac{10}{x} - 1 \right]$
- b.  $h(x) = 3(5x - 2)e^x$

**Exercise n°13**

Calculate the derivatives of:

- a.  $h(x) = \frac{\sqrt{x}}{x^2+3}$
- b.  $h(x) = \frac{x}{e^{x+7}}$

**Exercise n°14**

Use the Quotient Rule to show that:

$$\frac{d}{dx} [\sec x] = \sec x \cdot \tan x$$

**Exercise n°15**

Calculate the derivative of:

- a.  $h(x) = \cot^5(9x)$
- b.  $\ln(6x^3 - 5x + 4) - x^2$

**Exercise n° 16**

Use implicit differentiation to show that

$$\frac{d}{dx} [\operatorname{arccot} x] = -\frac{1}{x^2 + 1}$$

**Exercise n°17**

Use  $\frac{d}{dx} [\operatorname{arcsec} x] = \frac{1}{|x|\sqrt{x^2-1}}$  to calculate the derivative of  $h(x) = 4 \cdot \operatorname{arcsec} x^3$

**Exercise n°18**

Find a formula for the  $n^{\text{th}}$  derivative of

- a.  $f(x) = \frac{1}{x}$
- b.  $f(x) = e^{2x}$

**Exercise n° 19**

Use logarithmic differentiation to compute the derivative of  $f(x) = x^x$