TP-1

Measurements and Uncertainties

Determination of the spring constant K and estimation of its uncertainty δK

Purpose of the TP

This work aims to learn some basic rules for estimating error limits and thus enhance the value of our measurements and numerical results.

The determination of the spring constant K of spring and the estimation of its uncertainty δK.

Absolute Uncertainty and Relative Uncertainty

Every experimental measurement is affected by an **error** whose value cannot be precisely determined. Nevertheless, even though it's impossible to determine the exact value of the error committed, it is possible for each type of error to calculate its upper limit (in absolute value), which we will call **absolute uncertainty**.

Absolute Uncertainty

The value of a physical quantity **g**, must always be accompanied by an uncertainty **δg**.
We will then write that:

$$\mathbf{g} = \bar{\mathbf{g}} \pm \delta\mathbf{g}$$

$\bar{\mathbf{g}}$: is the average, measured, or calculated value.

Example

$d = 366 \pm 2$ km ⇔ 364 km $< d <$ 368 km
$m = 2.58 \pm 0.03$ kg ⇔ 2.55 kg $< m <$ 2.61 kg

Relative Uncertainty

An absolute uncertainty does not provide insight into the quality of a measurement. That's why it's necessary to define relative uncertainty; it allows for estimating the precision of the obtained result.

$$\textit{Relative Uncertainty} = \frac{\textit{Absolute Uncertaity}}{\textit{Average Value}}$$

Relative uncertainty has no units; it is generally expressed as a percentage (%) or per mille (‰).

Example :

If m = (25,4 ± 0,2) m, then the relative uncertainty is: $\delta m/m = 0.2/25.4 = 0.8\%$.
If L = (6130 ± 40) cm, then the relative uncertainty is: $\delta L/L = 0.0065 = 0.65\%$.

Methods of Uncertainty Calculation

Direct Measurement

Regardless of the measured physical quantity x

$$x = \bar{x} \pm \delta x \quad \text{with}$$

$\bar{x}$: the average value. $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$

δx: absolute error. $\delta x = \sqrt{\delta x_{sys}^2 + \delta x_{ale}^2}$

δx_{sys}: The systematic error provided by the manufacturer.

δx_{ale}: Random error.

$$\delta x_{ale} = \frac{1}{N}\sqrt{\sum (x_i - \bar{x})^2}$$

Indirect Measurement

$q = \bar{q} + \delta q$ / $q = f(x_1, x_2, \ldots\ldots\ldots x_n)$

x_i: of measurable physical quantities from a direct or indirect function.

$$\bar{q} = f(\overline{x_1}, \overline{x_2}, \ldots\ldots\ldots\ldots\ldots\ldots .\overline{x_n})$$

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x_1}\cdot \delta x_1\right)^2 + \left(\frac{\partial q}{\partial x_2}\cdot \delta x_2\right)^2 + \cdots\cdots\cdots + \left(\frac{\partial q}{\partial x_n}\cdot \delta x_n\right)^2}$$

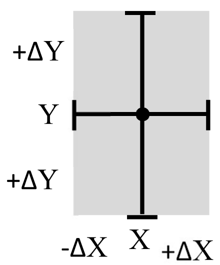
Example :

Here is the expression for velocity ***v***: $v = \frac{x}{t}$

$$\delta v = \sqrt{\left(\frac{\partial v}{\partial x}\cdot \delta x\right)^2 + \left(\frac{\partial v}{\partial t}\cdot \delta t\right)^2} \Rightarrow \delta v = \sqrt{\left(\frac{1}{t}\cdot \delta x\right)^2 + \left(-\frac{x}{t^2}\cdot \delta t\right)^2} \Rightarrow$$

The uncertainties on the graph

It is common to study graphically a property as a function of a parameter in order to deduce or verify a **law** connecting them.



The **uncertainty rectangles** (or **error bars**) should be plotted on the graph to assess the validity of the interpretation.

Consider an experimental point defined by the coordinates: X with an uncertainty of ± δX, and Y with an uncertainty of ± δY. The plotting of this point on a graph corresponds to the diagram shown below.

The hatched area corresponds to the uncertainty area of the experimental point. It can be reduced to a simple bar if one of the uncertainties is very small (then referred to as *error bars*).

Once the error rectangles are placed, you manually draw the best-fitting curve that *passes through all the uncertainty rectangles.*

Plotting the graph

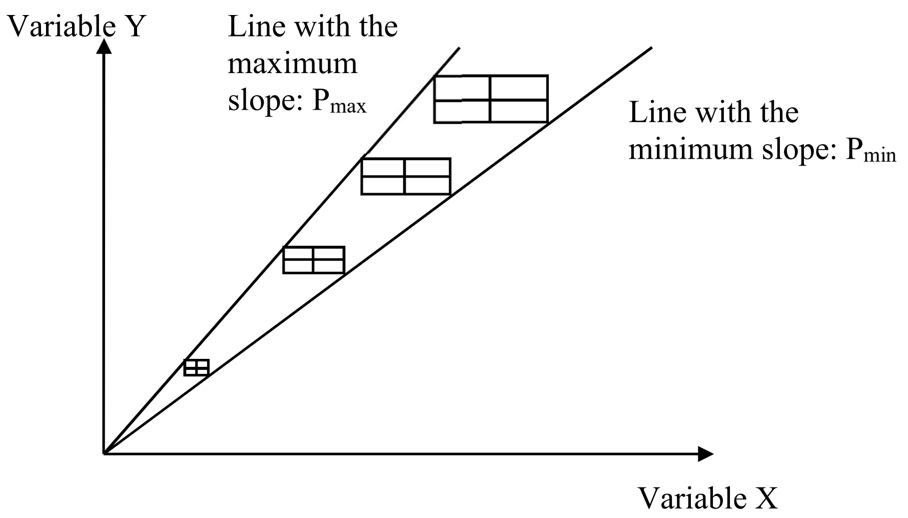
To obtain, within a reasonable time, a usable graph where it is easy to analyze a phenomenon, it is recommended to follow the following nine steps:

1- Carefully perform the various measurements and record the results in a table.
2- Estimate the uncertainties δx and δy for each pair (x, y) in the table.
3- Choose the origin of the axes appropriately (it's not necessary for the origin of the axes to correspond to x = 0 and y = 0).
4- Label the axes indicating the units of x and y.
5- Choose an appropriate scale for each of the two axes (experimental points should be distributed over a large part of the paper used).
6- Indiquate on each axis, according to the scale, a few points corresponding to integer values forming an arithmetic progression. (The values from the table should not appear on the axes).
7- Represent the experimental points with crosses (+).
8- Represent the uncertainty rectangles with sides of 2δx and 2δy (it's possible that the uncertainty on one axis is negligible, in which case the *uncertainty rectangles* become *error bars*).
9- Draw the curve Y(x) which should:

- intersect all the uncertainty rectangles.
- Have a continuously varying slope (**no broken lines or zigzags**).
- If Y(x) is **a straight line**, then there is a whole bundle of lines passing through all the uncertainty rectangles. In that case, you should represent two lines: one with **the minimal slope** and the other with **the maximal slope**.

The slope (P) and its uncertainty (ΔP) will be written as**:** $p_e = \bar{p}e \pm \delta p_e$

$$p_e = \frac{p_{emax} + p_{emin}}{2} \pm \frac{|p_{emax} - p_{emin}|}{2}$$



Indicative Graph

Manipulation

Determination of the stiffness constant K of a spring and estimation of its uncertainty δK

Experimental device

- A stand equipped with a **1-meter**-long ruler. and a systematic error $\delta x_{sys} = 1\ mm$.
- A spring (with a resting length $l_o = 100\ mm$).
- Six masses ranging from 200 g à 700 g ($\delta M = 8\ g$)

Work to be done

1- Attach the spring to the stand and measure its unloaded length l_o.
2- Attach a mass **m** to the spring, wait for the equilibrium state, and then measure the new length **l** of the spring. Deduce the elongation $x = (l - l_o)$ of the spring.
3- Successively assign the mass **m** values ranging from 200g to 700g and record the corresponding values of **x**.

Note: For each value of **m**, three measurements of x are necessary. Then, the average value of x and its uncertainty δx are deduced. At the end, write the average value of x and its uncertainty δx in the table.

4- Put your measurement results in table 1.

m(kg)	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**
x (m)						
δx(m)						

Table1: The elongation x as a function of the mass **m**.

5- Plot the graph representing the variation of x as a function of m, indicating the uncertainty rectangles (assuming that the masses used in the lab are known with an uncertainty of **± 8g**).

6- Indicate on the graph the maximum and minimum slopes, then deduce the average slope value $\bar{p}_e$ and its uncertainty δp_e.

7- Demonstrate that the slope :

$$p_e = \frac{g}{K}$$

g : gravity acceleration.
K : spring constant.

8- Knowing that **g=9.80 ±0.02 m/s²**, deduce the stiffness constant **K** of the spring.

9- Express the uncertainty δK as a function of δp_e et δg, using an uncertainty calculation using the logarithmic method.

10- Present the final result correctly (i.e., the value of K, its uncertainty, and its precision **δK**).